Queens School

Physics Department

AS Bridging Workbook

Name:______________________

Chapter 1: Rearranging equations

The first step in learning to manipulate an equation is your ability to see how it is done once and then repeat the process again and again until it becomes second nature to you.

In order to show the process once I will be using letters rather than physical concepts.

You can rearrange an equation $a = b \times c$ with

b as the subject $b = \frac{a}{c}$

or c as the subject $c = \frac{a}{b}$

Any of these three symbols a, b, c can be itself a summation, a subtraction, a multiplication, a division, or a combination of all. So, when you see a more complicated equation, try to identify its three individual parts a, b, c before you start rearranging it.

Worked examples

Equation	First Rearrangement	Second Rearrangement
$v = f \times \lambda$	$f = \frac{v}{\lambda}$	$\lambda = \frac{v}{f}$
$T = \frac{1}{f}$	$1 = T \times f$	$f = \frac{1}{T}$
$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$	$1 = v \times \left(\frac{1}{u} + \frac{1}{f}\right)$	$v = \frac{1}{\frac{1}{u} + \frac{1}{f}}$

THINK! As you can see from the third worked example, not all rearrangements are useful. In fact, for the lens equation only the second rearrangement can be useful in problems. So, in order to improve your critical thinking and know which rearrangement is the most useful in every situation, you must practise with as many equations as you can.

NOW TRY THIS!

From now on the multiplication sign will not be shown, so $a = b \times c$ will be simply written as $a = bc$ Fill in the blank spaces with your own examples.

Equation	First Rearrangement	Second Rearrangement
(Power of lens) $P = \frac{1}{f}$	$1 =$	$f =$
(Magnification of lens) $m = \frac{v}{u}$	$v =$	$u =$
(refractive index) $n = \frac{c}{v}$	$c =$	$v =$
(current) $I = \frac{\Delta Q}{\Delta t}$		
(electric potential) $V = \frac{\Delta E}{\Delta Q}$		
(power) $P = \frac{\Delta E}{\Delta t}$		
(power) $P = VI$		
(conductance) $G = \frac{I}{V}$		
(resistance) $R = \frac{V}{I}$		
(resistance) $R = \frac{1}{G}$		
(power) $P = I^2 R$		
(power) $P = \frac{V^2}{R}$		
(stress) $\sigma = \frac{F}{A}$	$F =$	$A =$
(strain) $\varepsilon = \frac{x}{l}$	$x =$	$l =$
(Young's modulus) $E = \frac{\sigma}{\varepsilon}$	$\sigma =$	$\varepsilon =$
(conductance) $G = \frac{\sigma A}{L}$		
(resistance) $R = \frac{\rho L}{A}$		
(resistivity) $\rho = \frac{1}{\sigma}$		

(phase angle) $\theta = 2\pi ft$	$f =$	$t =$	
(displacement) $y = a\sin\theta$	$a =$	$\sin\theta =$	
(Young's interference) $x = \frac{\lambda L}{d}$			
(quantum energy) $E = hf$	$f =$	$h =$	
(electron wavelength) $\lambda = \frac{h}{mv}$			

Chapter 2: Ratios

2.1 Manipulating ratios

These are the most useful ways of manipulating ratios. They will help you when you rearrange equations.

First: $\frac{A}{B} = \frac{C}{D} \Leftrightarrow A \times D = B \times C$

Second: $\frac{A}{B} = \frac{C}{D} \Leftrightarrow \frac{B}{A} = \frac{D}{C}$

Third: $\frac{A}{B} = \frac{C}{D} \Leftrightarrow \frac{A+B}{B} = \frac{C+D}{D}$

Fourth: $\frac{A}{B} = \frac{C}{D} \Leftrightarrow \frac{A-B}{B} = \frac{C-D}{D}$

and finally, this is how you simplify a complex fraction: $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \times d}{b \times c}$

THINK!

It is always useful to convert a complex fraction into a simple one, using the form above, before you try anything else. If b or d are missing, substitute them with number 1.

NOW TRY THIS!

Here are some examples to do: [6]

$$\frac{\frac{1}{4}}{\frac{2}{3}} =$$

$$\frac{\frac{3}{7}}{100} = \frac{\frac{3}{7}}{\frac{100}{1}} =$$

$$\frac{11}{\frac{2}{3}} =$$

Molecules of DNA have been stretched using “optical tweezers”. Using the following data find the cross sectional area of a DNA strand, if the Young’s modulus of DNA is 10^8 Pa and a load of 4 x 10^{-10} N results in 20% strain. [1]

Here is how you start: manipulate the equation until you have *A* on the left-hand side and only at this point substitute all the symbols for their equivalent numbers.

$$E = \frac{\sigma}{\varepsilon} \Rightarrow E = \frac{\frac{F}{A}}{\varepsilon} \Rightarrow$$

(Ans: 2 x10^{-17} m^2)

2.2 How to convert “sentences” into ratios

Have a look at these problems and try to solve them.

Problem 1

Anna has 800 apples in baskets. Each basket holds 16 apples. How many baskets does she have?

(Ans: 50 baskets)

Problem 2

The electric current through a wire is 3 A (3 Coulombs per second). Each electron has a charge of 1.6×10^{-19} C. How many electrons pass through the wire per second?

(Ans:1.88×10^{19} electrons)

Problem 3

A swimming pool is 27 m long. A transverse wave of 3 m wavelength is created. How many complete oscillations are created if the wave fills the pool?

(Ans: 9 waves)

THINK! Did you find problem 2 more difficult to do? If yes, think about this: all the problems follow the same logical sequence. Baskets (in problem 1) correspond to electrons (in problem 2) and wavelengths (in problem 3). Apples (in problem 1) correspond to charge (in problem 2) and length (in problem 3).

Here is another set of problems:

Problem 4

John has 147 pears in 21 baskets. How many baskets does he need for 14 pears?

(Ans: 2 baskets)

Problem 5

The weight of a 50.0 kg person on the moon is 80.0N. How much would a 72.0 kg person weigh on the moon?

(Ans: 115 N)

Problem 6

When stereo sound information is transmitted through a cable, 32 bits are sent every 22.7μs. Calculate how many bits you can send during 2 seconds (2 s = 2 x 10^6 μs)

(Ans: 2.8 x 10^6 bits)

THINK! Did you find problem 6 more difficult to do? If yes, think about this: all the problems follow the same logical sequence. Baskets (in problem 4) correspond to weight (in problem 5) and bits (in problem 6). Pears (in problem 4) correspond to mass (in problem 5) and time (in problem 6).

FOLLOW THIS! You can solve all the above problems and many more using the method below:

STEP 1. Write two sentences, one exactly below the other, that describe the relationship between the two variables. Make sure you have the same variable on each side. Use the letter "x" for the unknown variable.

e.g. ***8*** bits in ***1*** byte
3200 bits in ***x*** byte

STEP 2. Convert the two sentences into a ratio

e.g. $$\frac{8}{3200} = \frac{1}{x}$$

STEP 3. Use the properties of ratios to help you rearrange and solve the problem.

e.g. $$8x = 3200 \Rightarrow x = \frac{3200}{8} \Rightarrow x = 400$$

NOW TRY THIS!

Convert the degrees to radians and the radians to degrees using ratios:

e.g. ***180*** degrees in ***π*** radians
x degrees in **y** radians

Degrees	Radians
42	
64	
170	
	2
	4
	6

Chapter 3: How to use and convert prefixes

Mathematical Prefixes[2]

Prefix	Symbol	Name	Multiplier
femto	f	quadrillionth	10^{-15}
pico	p	trillionth	10^{-12}
nano	n	billionth	10^{-9}
micro	µ	millionth	10^{-6}
milli	m	thousandth	10^{-3}
centi	c	hundredth	10^{-2}
deci	d	tenth	10^{-1}
deka	da	ten	10^{1}
hecto	h	hundred	10^{2}
kilo	k	thousand	10^{3}
mega	M	million	10^{6}
giga	G	billion†	10^{9}
tera	T	trillion†	10^{12}
peta	P	quadrillion	10^{15}

When you are given a variable with a prefix you must convert it into its numerical equivalent in standard form before you use it in an equation.

FOLLOW THIS! Always start by replacing the prefix symbol with its equivalent multiplier.

For example: 0.16 µA = 0.16 x 10^{-6} A

3 km = 3 x 10^{3} m

10 ns = 10 x 10^{-9} s

DO NOT get tempted to follow this further (for example: 0.16 x 10^{-6} A = 1.6 x 10^{-7} A and also 10 x 10^{-9} s = 10^{-8} s) unless you are absolutely confident that you will do it correctly. It is always safer to stop at the first step (10 x 10^{-9} s) and type it like this into your calculator.

NOW TRY THIS!

1.4 kW =
10 µC =
24 cm =
340 MW =
46 pF =
0.03 mA =
52 Gbytes =
43 kΩ =
0.03 MN =

Chapter 4: How to solve a lengthy problem

Problems in physics can appear to be difficult at first sight. However, once you analyse the problem in well-defined steps you should be able to solve it without any difficulty. The steps you need to follow are:

1. **Identify the variables** you are given and the ones you are asked to find
2. **Convert all units** given to SI units
3. **Give a different symbol** to each variable; try to stick to the well known symbols. To simplify, write the values for each symbol (*m=600*) but don't worry about writing the units at this stage.
4. **Recognise which equation/s** to use. You do this by looking at what variables are available to you and what variables you are asked to find. This is a critical stage; experience is the most important factor here. This is why you need to practise again and again... This is also why you need to KNOW ALL YOUR EQUATIONS VERY WELL!
5. **Find the logical sequence** for using these equations in order to reach the desirable outcome. Again, experience is very important here!
6. Write the final answer and **add the correct units.**

EXAMPLE

A car of mass 600 kg is travelling at 10 ms^{-1}. When the brakes are applied, it comes to rest in 0.01 km. What is the average force exerted by the brakes? [3] (Note: there are many ways to solve this problem and in A2 a shorter method will be introduced.)

STEP 1. mass= 600 kg

initial velocity = 10 m s^{-1}

final velocity = 0 m s^{-1}

stopping distance = 0.01 km

force = ?

STEP 2. Are they all in SI units? No. So ...

stopping distance = 0.01 km = 0.01 x 10^3 m = 10 m

STEP 3. $m = 600$

$u = 10$

$v = 0$

$s = 10$

$F = ?$

STEP 4. $\Delta E = W$ (Change in energy = work done)

$W = Fs$ (Work done = force x distance in the direction of the force)

$E_K = \frac{1}{2}mv^2$ (Kinetic energy = 0.5 x mass x velocity x velocity)

STEP 5. Find the initial kinetic energy:

$$E_K = \frac{1}{2}mu^2 = \frac{1}{2} \times 600 \times 10^2 = 30000$$

Find the final kinetic energy:

$$E_K{}' = \frac{1}{2}mv^2 = \frac{1}{2} \times 600 \times 0^2 = 0$$

Find the change in kinetic energy: $\Delta E = E_K{}' - E_K = 0 - 30000 = -30000$

Equalise the change in kinetic energy with the work done and rearrange to find the force:

$$\Delta E = W \Rightarrow \Delta E = Fs \Rightarrow F = \frac{\Delta E}{s} \Rightarrow$$

$$\Rightarrow F = \frac{-30000}{10} \Rightarrow F = -3000$$

STEP 6. The final answer is $F = -3000N$

The negative sign shows that the force is in opposite direction to the velocity, i.e. it is the frictional force that stops the car.

NOW TRY THIS!

A girl diving from a 15 m platform wishes to know how fast she enters the water. She is in the air for 1.75 s and dives from rest (with an initial speed of zero). What can you tell her about her entry speed? (g = 9.8 m s^{-2}) [7]

REMEMBER! Follow all steps! Do not try to rush through!

STEP 1.

STEP 2.

STEP 3.

STEP 4.

STEP 5.

STEP 6.

Chapter 5: How to recognise hidden mathematical meanings in sentences

This chapter gives you a first taste of the hidden mathematical meanings in sentences and words. Add your own as you progress in the course.

SENTENCE	MATHEMATICAL MEANING
Charge ***per*** unit time	Charge **divided** by time
Metres ***per*** second per second	Metres **divided** by second squared
Resistance ***per*** unit length	Resistance **divided** by length
the ***product*** of voltage and current	**Multiply** voltage with current
The ***sum*** of powers of the lenses	**Add** all powers of lenses

Chapter 6: Logarithms

6.1 Definition of logarithms

The use of logarithms sounds really scary but once you learn the basics you will be able to manipulate equations with logarithms with no problem.

This is the most important equation that summarises the idea behind all logarithms: [4]

$$a^x = y \Leftrightarrow \log_a y = x$$

and here are a few examples:

$2^3 = 8 \Leftrightarrow \log_2 8 = 3$ (you read: the logarithm of 8 to the base 2 is 3)

$10^3 = 1000 \Leftrightarrow \log_{10} 1000 = 3$ (read: the log of 1000 to the base 10 is 3)

NOW TRY THIS!

$... = ... \Leftrightarrow \log_3 9 = 2$

$10^{1.699} = 50 \Leftrightarrow ... = ...$

$... = ... \Leftrightarrow \log_5 0.2 = -1$

$... = ... \Leftrightarrow \log_3 27 = ...$

$... = ... \Leftrightarrow \log_2 64 = ...$

$... = ... \Leftrightarrow \log_4 64 = ...$

$... = ... \Leftrightarrow \log_8 64 = ...$

$... = ... \Leftrightarrow \log_5 625 = ...$

6.2 Properties of logarithms

All the relationships below are true, regardless of the value of the base 'a'. [5]

$\log_a 1 = 0$

$\log_a a = 1$

$$\log_a(p \times q) = \log_a p + \log_a q$$

$$\log_a\left(\frac{p}{q}\right) = \log_a p - \log_a q$$

$$\log_a(x^r) = r\log_a x$$

6.3 The two most famous logarithms

In physics the two most useful logarithms are:

- the Common logarithms which use base 10 and
- the Natural logarithms which use the irrational number *e (e= 2.71828183)* as the base.

To avoid complicating the equations, the symbol for Common logarithms is $\log x$ instead of $\log_{10} x$ and the symbol for Natural logarithms is $\ln x$

6.4 How to solve problems with logarithms

There are many areas of physics where you will encounter logarithms: sound levels, noise limitation on maximum bits per sample, radioactive decay, discharge of capacitor etc.

The important thing is to know what to do in all cases.

When you are asked to manipulate a logarithmic equation remember to **USE the definition of a logarithm as part of the manipulation process**.

For instance, if you want to solve the equation below with V_{total} as the subject you follow these steps:

$$b = \log_2\left(\frac{V_{\text{total}}}{V_{\text{noise}}}\right) \Leftrightarrow 2^b = \frac{V_{\text{total}}}{V_{\text{noise}}} \Rightarrow V_{\text{total}} = 2^b V_{\text{noise}}$$

NOW TRY THIS!

1. Solve the same equation, but with V_{noise} as the subject:

2. The number of decibels d is given by the equation

$$d = 10\log_{10}\frac{I_2}{I_1}$$

a) Rearrange this equation with I_2 as the subject (the first step has been done for you):

$$d = 10\log_{10}\frac{I_2}{I_1} \Rightarrow \frac{d}{10} = \log_{10}\frac{I_2}{I_1} \Rightarrow$$

b) Now, rearrange this equation with I_1 as the subject:

3. A sound that is on the threshold of audible intensity is 10^{-12} W m^{-2} (like hearing a pin drop). This is taken as the baseline intensity I_1. A painful sound (like a jet taking off) has an intensity of about 10 W m^{-2}. What is the sound level of the jet in dB? [8]. The decibel formula is given in question 2 above.

(Ans: 130 dB)

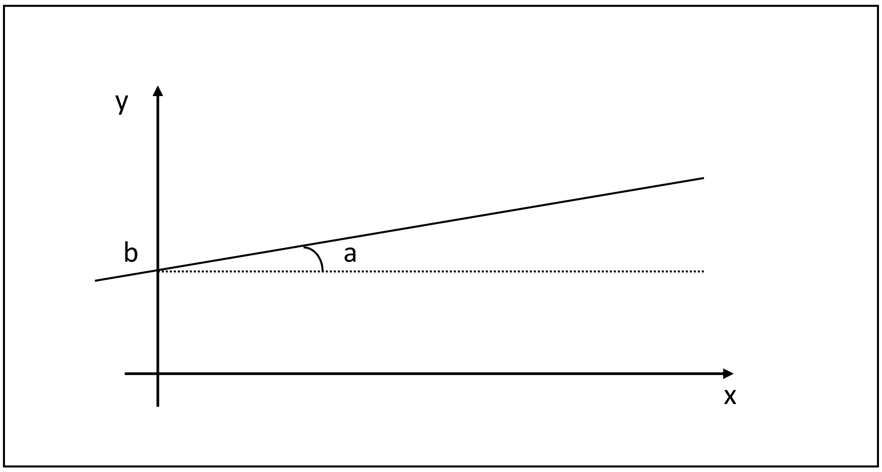
Chapter 7: Graphs

7.1 Types of graphs

Graphs are VERY important in physics because they show patterns between variables. A straight line graph that starts from the (0,0) point is the best proof that two variables are directly proportional.

Straight line graph

You should know from your maths that the general equation for a straight line is $y = ax + b$, where a is the gradient of the graph and b is the point that the line cuts the y-axis.



You must also know from your maths the equation for a hyperbola, a parabola, an ellipse etc. Make your own table including all the graph shapes you know and their functions. Make sure you include the logarithmic and exponential functions.

Function	Graph	Example in physics
$y = ax + b$		

7.2 How to choose the right graph for plotting

Although the above list is important, when it comes to finding a relationship between two variables the only graph that can show this very clearly is the straight line graph.

EXAMPLE

Let's say that you want to prove the relationship between the kinetic energy of an object and its velocity. You plot velocity on the x-axis and kinetic energy on the y-axis. You will get a curve which as you now is a parabola (since the kinetic energy is directly proportional to the square of the velocity).

Now let's say you do another experiment that, unknown to you, also follows the same pattern. You will also get a curve when you plot the graph. Will you be able to recognise that this is a parabola? What if it is a curve that is very close to a parabola but not quite?

What can you do to be sure that you have cracked the relationship?

Think again about the example above. If instead of plotting kinetic energy against velocity you plot kinetic energy against velocity **squared** what will you get? You will get <u>a straight line through zero!</u> Moreover, you will be certain that the relationship is that: <u>the kinetic energy is directly proportional to the velocity squared.</u>

So what have we learned so far?

ALWAYS AIM AT PLOTTING TWO VARIABLES THAT WILL GIVE YOU A STRAIGHT LINE!

Here are some examples:

- To prove that resistance R is inversely proportional to cross sectional area A, plot R against $\frac{1}{A}$. This should give you a straight line.
- To prove that the square of the period T of a pendulum is directly proportional to its length l plot either T^2 against l or T against $\sqrt{l}$

NOW TRY THIS!

1. The pendulum equation is:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

a) What variables should you plot against each other in order to prove that the period of the pendulum does not depend on its mass? What will the shape of this graph be?

b) What variables should you plot against each other to prove that the period depends on the gravitational field strength as shown by the equation?

2. The universal gravitational law is given by the equation:

$$F = -G\frac{mM}{r^2}$$

a) What variables should you plot against each other in order to prove that the attractive force is directly proportional to both masses of the objects?

b) What variables should you plot against each other in order to prove that the attractive force is inversely proportional to the distance squared between the objects?

7.3The significance of the gradient

During your course you will be asked to decide which graphs to plot in order to show a relationship or to calculate a physical constant.

We have already noted how important it is to aim at plotting a graph that will end up being a straight line. This gives you a definite answer about the relationship between the two variables. But there is more to it. The gradient of this line will give you information about a constant in your experiment.

EXAMPLE

Let's say that you want to measure the gravitational field strength of Earth with a pendulum. You vary the length and measure the period. You then decide to plot T^2 against l . The graph will be a straight line. What will its gradient be? To find this, compare the pendulum equation with the straight line equation as shown below:

$$T^2 = 4\pi^2\frac{l}{g}$$

$$y = ax + b$$

I hope you can see that y corresponds to T^2, x corresponds to l , b corresponds to zero, and a corresponds to $\frac{4\pi^2}{g}$. This tells you that once you measure the gradient from your graph you will know the value of $\frac{4\pi^2}{g}$ and you will then be able to calculate g from this as:

$$gradient = \frac{4\pi^2}{g} \Rightarrow g = \frac{4\pi^2}{gradient}$$

NOW TRY THIS!

Try to find the gradient in all the situations listed below. The first three have been done for you.

Equation	Plot y against x	gradient	Constant
$R = \frac{V}{I}$	$x - axis: current$ $y - axis: voltage$	$gradient = R$	R (for a fixed resistor)
$R = \frac{V}{I}$	$x - axis: voltage$ $y - axis: current$	$gradient = \frac{1}{R}$	R (for a fixed resistor)
$E = \frac{\frac{F}{A}}{\frac{x}{l}}$	$x - axis: force$ $y - axis: extension$	$gradient = \frac{l}{EA}$	$E = \frac{l}{A \times gradient}$ (Young's modulus)
$R = \frac{\rho L}{A}$	$x - axis: L$ $y - axis: R$	$gradient =$	$\rho =$ (resistivity)
$R = \frac{\rho L}{A}$	$x - axis: \frac{1}{A}$ $y - axis: R$	$gradient =$	$\rho =$ (resistivity)
$\frac{1}{2}mv^2 = Fs$ (stopping distance-velocity relationship)	$x - axis: v^2$ $y - axis: s$	$gradient =$	$F =$ (fricition)
$xd = \lambda L$ (double slit interference)	$x - axis: \frac{1}{d}$ $y - axis: x$	$gradient =$	$\lambda =$ (wavelength)
$xd = \lambda L$ (double slit interference)	$x - axis: L$ $y - axis: x$	$gradient =$	$\lambda =$ (wavelength)

Apart from its use as explained above, the gradient in all lines (curved or straight) corresponds to the **derivative** of the function you plot. This is why if you plot time on the x-axis and displacement on the y-axis the gradient corresponds to the velocity of the object. If the line is curved the gradient does not stay the same, which means that it is equal to the instantaneous velocity of the object.

For the same reason if you plot time on the x-axis and velocity on the y-axis the gradient corresponds to the acceleration of the object. If the line is curved the gradient does not stay the same, which means that it is equal to the instantaneous acceleration of the object.

7.4The significance of the area under a graph

The area between a graph of $y = f(x)$ and the x-axis is equal to the **definite integral** of the function. This formula gives a positive result for a graph above the x-axis, and a negative result for a graph below the x-axis. [9]

This is why the area under a velocity-time graph is equal to the distance covered by the object.

If the graph is a straight line then the area under can be calculated very precisely as the area of a triangle or trapezium etc. If the line is a curve, the area is often estimated to a good precision before it can give you some useful information.

You will use this idea more at A2 level so for the moment we will leave it at this stage.